Change of Variables Summary

- The double integral in polar coordinates
  - $\iint_R f(x,y)\,dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$

  AND $\iint_D f(x,y)\,dA = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$
  - Note that $dV = r\,dr\,d\theta$

- The triple integral in cylindrical coordinates
  - $\iiint_E f(x,y,z)\,dV = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} \int_{u_1(r\cos\theta, r\sin\theta)}^{u_2(r\cos\theta, r\sin\theta)} f(r\cos\theta, r\sin\theta)\,r\,dz\,dr\,d\theta$
  - Note that $dV = r\,dz\,dr\,d\theta$
  - Typically used when you see x^2+y^2, when D is circular, or when the integrand is easier in cylindrical coordinates

- The triple integral in spherical coordinates
  - $\iiint_E f(x,y,z)\,dV$
    $$= \int_c^d \int_\alpha^\beta \int_{g_1(\theta,\phi)}^{g_2(\theta,\phi)} f(\rho\cos\theta\sin\phi, \rho\sin\theta\sin\phi, \rho\cos\phi)\,\rho^2\sin\phi\;d\rho\,d\theta\,d\phi$$
  - Note that $dV = \rho^2\sin\phi\;d\rho\,d\theta\,d\phi$
  - Typically used when you see $x^2+y^2+z^2$ or when E involves cones/spheres

- Change of variables for double integrals (under $T: S \to R$)
  - $\iint_R f(x,y)\,dA = \iint_S f\big(g(u,v), h(u,v)\big)\left|\frac{\partial(x,y)}{\partial(u,v)}\right|\,du\,dv$, where
    $$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix}$$
  - $u = f(x,y)$ and $v = g(x,y)$ are useful in finding limits of integration
  - $x = x(u,v)$ and $y = y(u,v)$ are useful for finding $\frac{\partial(x,y)}{\partial(u,v)}$
  - Substitute using whichever is most convenient